B.sc(H) part1 paper 1

Topic: Algebric Laws for Multiplication of Matrices

Subject:Mathematics

Dr Hari Kant Singh

RRS college ,Mokama

Algebric laws of multiplication of matrices

Associative law : *If A and B are conformal for the product AB and B and C are conformal for the product BC, then*

$$(AB)C = A(BC),$$

Proof : Let A, B, C be the $m \times n$, $n \times p$ and $p \times q$ matrices and let $A = [a_{ij}]$, $B = [b_{ij}]$, $C = [c_{ij}]$.

Here A, B and C are conformal for the product AB and BC.

Now $(AB) = [a_{ij}] \times [b_{ij}] = \left[\sum_{k=1}^{n} a_{ik} b_{kj} \right]$ ...(I)

$= [\lambda_{ij}]$, say $i = 1, 2, 3, \ldots m$

$j = 1, 2, 3, \ldots p$.

We find that (AB) i.e., $[\lambda_{ij}]$ is a $m \times p$ matrix and since C is a $p \times q$ matrix; therefore (AB) and C are conformal for the product $(AB)C$ and $(AB)C$ is a $m \times q$ matrix.

Hence $(AB)C = [\lambda_{ij}] \times [c_{ij}]$

$$= \left[\sum_{l=1}^{p} \lambda_{il} c_{lj} \right] = \left[\sum_{l=1}^{p} \left(\sum_{k=1}^{n} a_{ik} b_{kl} \right) c_{lj} \right]; \text{ from (I)}$$

$$= \left[\sum_{l=1}^{p} \sum_{k=1}^{n} a_{ik} b_{kl} c_{lj} \right]; \quad i = 1, 2, 3, \ldots m; \quad j = 1, 2, 3, \ldots q.$$

Again $(BC) = [b_{ij}] \times [c_{ij}] = \left[\sum_{r=1}^{p} b_{ir} c_{rj} \right]$ ...(II)

$= [\delta_{ij}]$. say; $i = 1, 2, 3, \ldots n; j = 1, 2, 3, \ldots q$.

We find that (BC) i.e., $[\delta_{ij}]$ is a $n \times q$ matrix and since A is a $m \times n$ matrix, therefore A and (BC) are conformal for the product $A(BC)$ and $A(BC)$ is a $m \times q$ matrix.

Hence $A(BC) = [a_{ij}] \times [\delta_{ij}]$

$$= \left[\sum_{s=1}^{n} a_{is} \delta_{sj} \right] = \left[\sum_{s=1}^{n} a_{is} \left(\sum_{r=1}^{p} b_{sr} c_{rj} \right) \right]; \text{ from (II)}$$

$$= \left[\sum_{r=1}^{p} \sum_{s=1}^{n} a_{is} b_{sr} c_{rj} \right]; \quad i = 1, 2, 3, \ldots m; \quad j = 1, 2, 3, \ldots q.$$

Thus $(AB)C = A(BC)$.

We may write $(AB)C = A(BC) = ABC$.

[illegible]) Distributive law : *If A and B are conformal for the product AB, B and C are conformal for addition, then* $A(B + C) = AB + AC$

Proof : Let A, B, C be the $m \times n$, $n \times p$ and $n \times p$ matrices and let $A = [a_{ij}]$, $B = [b_{ij}]$, $C = [c_{ij}]$.

Since B and C are conformal,

$$\therefore \quad B + C = [b_{ij}] + [c_{ij}] = [b_{ij} + c_{ij}].$$

Now, $B + C$ is a $n \times p$ matrix. Therefore A and $B + C$ are conformal for the product $A(B + C)$.

Hence $A(B + C) = [a_{ij}] \times [b_{ij} + c_{ij}]$

$$= \left[\sum_{k=1}^{n} a_{ik}\,(b_{kj} + c_{kj})\right]; \; i = 1, 2, 3, \ldots m, \quad j = 1, 2, 3, \ldots p$$

$$= \left[\sum_{k=1}^{n} a_{ik}\, b_{kj} + \sum_{k=1}^{n} a_{ik}\, c_{kj}\right]$$

$$= \left[\sum_{k=1}^{n} a_{ik}\, b_{kj}\right] + \left[\sum_{k=1}^{n} a_{ik}\, c_{kj}\right] \qquad \ldots(1)$$

But $AB = [a_{ij}] \times [b_{ij}] = \left[\sum_{k=1}^{n} a_{ik}\, b_{kj}\right]; \; i = 1, 2, 3, \ldots m, \quad j = 1, 2, 3, \ldots p$

and $AC = [a_{ij}] \times [c_{ij}] = \left[\sum_{k=1}^{n} a_{ik}\, c_{kj}\right];$ " " $\qquad \ldots(2)$

Therefore from (1) and (2), we have $A(B + C) = AB + AC$.

Similarly, $(B + C)D = BD + CD$, when D is a $p \times q$ matrix (say)